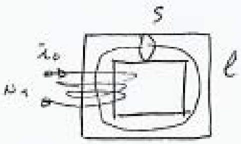
Transformateur d'intensité

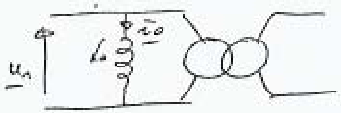


1) $\boxed{\mathcal{R} = \frac{1}{\mu}\frac{\ell}{S}}$ $\left(N_1 i_0 = H\ell = \frac{B}{\mu}\ell = \frac{\varphi}{\mu}\frac{\ell}{S} = \mathcal{R}\varphi \right)$

2) $\phi_t = N_1 \varphi = N_1 \frac{N_1 i_0}{\mathcal{R}} = \frac{N_1^2}{\mathcal{R}} i_0$

or $\phi_t = L_0 i_0$ donc $\boxed{L_0 = \frac{N_1^2}{\mathcal{R}} = N_1^2 \mu \frac{S}{\ell}}$

3) $N_1 \underline{i_1} + N_2 \underline{i_2} = N_1 \underline{i_0}$ (1) $\underline{u_2} = -\frac{N_2}{N_1}\underline{u_1}$



$\underline{u_1} = j L_0 \omega \underline{i_0}$

soit $\underline{i_0} = \frac{\underline{U_1}}{j L_0 \omega}$

$\underline{u_2} = \underline{Z_2}\,\underline{i_2} \longrightarrow \underline{i_2} = \frac{\underline{U_2}}{\underline{Z_2}} = -\frac{N_2}{N_1}\frac{\underline{U_1}}{\underline{Z_2}}$

$N_1 \underline{i_1} + N_2 \underline{i_2} = N_1 \underline{i_0}$

$\longrightarrow N_1 \underline{i_1} - \frac{N_2^2}{N_1}\frac{\underline{U_1}}{\underline{Z_2}} = \frac{N_1 \underline{U_1}}{j L_0 \omega}$

soit $N_1 \underline{i_1} = \underline{U_1}\left[\frac{N_1}{j L_0 \omega} + \frac{N_2^2}{N_1}\frac{1}{\underline{Z_2}}\right]$

donc $\boxed{\underline{U_1} = \frac{N_1 \underline{i_1}}{\frac{N_2^2}{N_1 \underline{Z_2}} + \frac{N_1}{j L_0 \omega}}}$

4) si on veut que $\underline{U_1}$ ne dépende que de $\underline{Z_2}$, il faut que $\frac{N_1}{L_0\omega} \ll \frac{N_2^2}{N_1 \|\underline{Z_2}\|}$

ou $\frac{L_0\omega}{N_1} \gg \frac{N_1}{N_2^2}\|\underline{Z_2}\|$

soit $\boxed{L_0\omega \gg \frac{N_1^2}{N_2^2}\|\underline{Z_2}\| = \|\underline{Z_2'}\|}$

$L_0\omega$ doit être très supérieur à l'impédance ramenée au primaire.

5) si $L_0\omega$ vérifie la condition précédente

alors $\underline{U_1} \simeq \frac{N_1^2}{N_2^2}\underline{Z_2}\,\underline{i_1} = \underline{Z_2'}\,\underline{i_1}$

si on veut que $\|\underline{U_1}\|$ soit très faible, quel que soit $\|\underline{i_1}\|$

il faut que $\underline{Z_2'}$ soit faible, donc que $\underline{Z_2}$ soit faible.

6) on a vu que $\underline{i_0} = \frac{\underline{U_1}}{j L_0 \omega}$

si $\underline{U_1} \to 0$ alors $\underline{i_0}$ devient très faible.

il faut par ailleurs que $L_0\omega$ soit grand

donc $\underline{i_0}$ est "doublement" faible.

7) on prend l'hypothèse $\|\underline{Z_2}\| \to 0$

et $\mu \to \infty$

si $\mu \to \infty$ alors $L_0 = \mu \frac{N_1^2 S}{\ell} \to \infty$

donc $L_0\omega \to \infty$

si $\|\underline{Z_2}\| \to 0$ alors $\|\underline{U_1}\| \to 0$

conclusion : si $\|\underline{U_1}\| \to 0$ et $L_0\omega \to \infty$, alors $\|\underline{i_0}\| \to 0$

comme $N_1 \underline{i_1} + N_2 \underline{i_2} = N_1 \underline{i_0}$ si $\underline{i_0} \to 0$

alors $N_1 \underline{i_1} = -N_2 \underline{i_2}$

soit $\boxed{\underline{i_2} = -\frac{N_1}{N_2}\underline{i_1} = -k\,\underline{i_1}}$ donc $\boxed{k = \frac{N_1}{N_2}}$

①

A) essai à vide.

1) machine au synchronisme $n_R = n_S$

$g = 0.$

2) $V = 220\,V$ $\quad P = 182\,W$ $\quad I = 0{,}96\,A.$

$P = 3\,V\,I\cos\varphi = \sqrt{3}\,U\,I\cos\varphi.$

$\cos\varphi = \dfrac{182}{3\times 220\times 0{,}96} = 0{,}2872. \quad \varphi = 73{,}3°$

soit $tg\,\varphi = 3{,}335$

$tg\,\varphi = \dfrac{Q}{P}$ soit $Q = P\,tg\,\varphi$

$Q = 607\ VAR.$

3) $p_{Ro} = \dfrac{V^2}{R_0} \qquad P_{Ro} = \dfrac{3V^2}{R_0}$

soit $R_0 = \dfrac{3V^2}{P_{Ro}} = \dfrac{3\times 220^2}{182} = 797{,}8\,\Omega.$

$q_{Lo} = \dfrac{V^2}{X_0} \qquad Q_{Lo} = \dfrac{3V^2}{X_0}$

soit $X_0 = \dfrac{3V^2}{Q_{Lo}} = \dfrac{3\times 220^2}{607} = 239{,}2\,\Omega$

$L_0 = \dfrac{X_0}{2\pi F} = \dfrac{239{,}2}{314} = 0{,}762\,H.$

B) essai à rotor bloqué.

1) rotor bloqué $n_R = 0$ soit $g = \dfrac{n_S - n_R}{n_S} = 1$

2) $V = 50\,V \quad P = 150\,W \quad I = 6{,}2\,A.$

$\cos\varphi = \dfrac{150}{3\times 50\times 6{,}2} = 0{,}1612 \qquad \varphi = 80{,}7°$

$tg\,\varphi = 6{,}119$

②

$Q = P\,tg\,\varphi = 150\times 6{,}119 = 917{,}8\ VAR.$

3) $P_{Ro} = \dfrac{3V^2}{R_0} = 9{,}4\,W. \ll 150\,W.$

$Q_{Lo} = \dfrac{3V^2}{X_0} = 31{,}3\ VAR \ll 917{,}8\ VAR$

4) $\underline{Z} = \dfrac{r}{g} + j\omega l = r + jx.$

$|\underline{Z}| = \sqrt{r^2 + x^2} = \dfrac{V}{I'} = \dfrac{50}{6{,}2} = 8{,}06\,\Omega.$

$tg\,\varphi = \dfrac{x}{r} = 6{,}119.$

5) $x = r\,tg\,\varphi. \qquad r = \dfrac{x}{tg\,\varphi}.$

$\dfrac{x^2}{tg^2\varphi} + x^2 = 8{,}06^2$

$x^2 = \dfrac{8{,}06^2}{1 + 1/tg^2\varphi} = \dfrac{8{,}06^2}{1{,}027}$

$x^2 = 63{,}26 \qquad x = 7{,}95\,\Omega.$

$r = \dfrac{7{,}95}{6{,}119} = 1{,}299\,\Omega$

soit $r = 1{,}3\,\Omega$

$l = \dfrac{7{,}95}{314} = 25{,}3\,mH.$

C) essai en charge.

1) $g = \dfrac{n_S - n_R}{n_S} = \dfrac{3000 - 2910}{3000} = \dfrac{90}{3000} = 0{,}03$

2) $\dfrac{r}{g} = \dfrac{1{,}3}{0{,}03} = 43{,}33\,\Omega.$

$$\underline{Z} = \frac{r}{g} + jx = 43{,}33 + 8j$$

$$|\underline{Z}| = \sqrt{43{,}33^2 + 8^2} = 44{,}06\,\Omega.$$

$$I' = \frac{220}{|\underline{Z}|} = 4{,}99\,A \simeq 5\,A.$$

$$tg\,\varphi' = \frac{x}{r/g} = \frac{8}{43{,}33} = 0{,}1846$$

$$\varphi' = 10{,}46°$$

3) $P_{abs} = P_{R_0} + P_{r/g}$

$$P_{R_0} = \frac{3V^2}{R_0} = 182W$$

$$P_{r/g} = 3\,\frac{r}{g}\,I'^2 = 3 \times 43{,}33 \times 5^2 = 3250\,W.$$

soit $P_{abs} = 3432\,W.$

4) $P_u = P_{r/g} - P_r = 3250 - \underbrace{3 \times 1{,}3 \times I'^2}_{97{,}5}$

$$P_u = 3152{,}5\,W.$$

$$P_u = C_u\,\omega \qquad C_u = \frac{3152{,}5}{2 \times \pi \times \underbrace{\frac{2910}{60}}_{48{,}5}} = 10{,}35\,mN.$$

$$\eta = \frac{P_u}{P_{abs}} = \frac{3152{,}5}{3432} = 0{,}919$$

5) $Q_{abs} = Q_{R_0} + Q_l$

$$Q_{L_0} = \frac{3V^2}{X_0} = \frac{3 \times 220^2}{240} = 605\,VAR.$$

$$Q_l = 3\,x\,I'^2 = 3 \times 8 \times 25 = 600\,VAR$$

soit $Q_{abs} = 1205\,VAR$

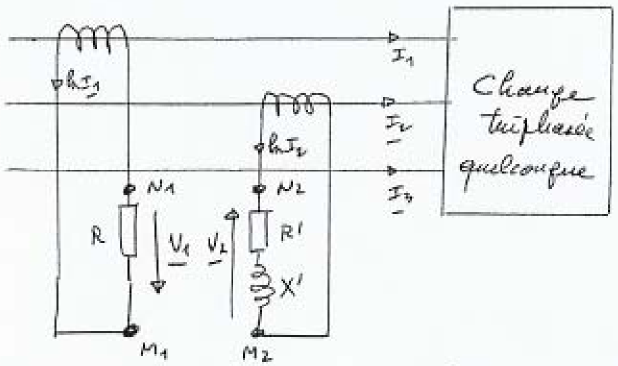
$$tg\,\varphi = \frac{Q_{abs}}{P_{abs}} = \frac{1205}{3432} = 0{,}3511$$

$$\varphi = 19{,}35°$$

$$\cos\varphi = 0{,}9435$$

$$I_{abs} = \frac{P_{abs}}{3V\cos\varphi} = \frac{3432}{3 \times 220 \times 0{,}9435} = 5{,}51\,A.$$

Mesure des composantes symétriques.



$$\begin{bmatrix} \underline{X_1} \\ \underline{X_2} \\ \underline{X_3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \underline{X_d} \\ \underline{X_i} \\ \underline{X_0} \end{bmatrix} \quad \text{et} \quad \begin{bmatrix} \underline{X_d} \\ \underline{X_i} \\ \underline{X_0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{X_1} \\ \underline{X_2} \\ \underline{X_3} \end{bmatrix}$$

$$a = e^{j\frac{2\pi}{3}}$$

$$\underline{Z} = R \qquad \underline{Z}' = R' + jX' \qquad X' = \sqrt{3}\,R' \qquad \|\underline{Z}'\| = R$$

1) $\underline{Z}' = R' + j\sqrt{3}\,R' = R'(1 + j\sqrt{3})$

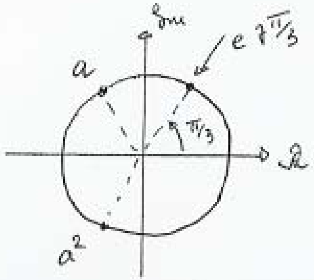
$= \rho e^{j\theta}$ on sait que $\rho = \|\underline{Z}'\| = R$

$\tan\theta = \dfrac{\sqrt{3}\,R'}{R'} = \sqrt{3}$

soit $\theta = \arctan\sqrt{3} = \pi/3$

donc $\boxed{\underline{Z}' = R\,e^{j\pi/3}}$

2)



alors $e^{j\pi/3} = -a^2$

donc $\boxed{\underline{Z}' = -a^2 R}$

3) $\underline{V_1} = h\,\underline{I_1}\,R$

$\underline{V_2} = h\,\underline{I_2}\,\underline{Z}' = -h\,\underline{I_2}\,a^2 R$

4) On relie N1 et N2

donc $\underline{V}_{M_1M_2} = \underline{V} = \underline{V_1} + \underline{V_2}$

soit $\underline{V} = h\,\underline{I_1}\,R - h\,\underline{I_2}\,a^2 R.$

5) neutre non relié.

alors $\underline{I_0} = \frac{1}{3}(\underline{I_1} + \underline{I_2} + \underline{I_3}) = 0$

6) on a montré que $\underline{V} = hR(\underline{I_1} - a^2\underline{I_2})$

on sait que $\underline{I_1} = \underline{I_d} + \underline{I_i} + \underline{I_0} = \underline{I_d} + \underline{I_i}$

$\underline{I_2} = a^2\underline{I_d} + a\,\underline{I_i} + \underline{I_0} = a^2\underline{I_d} + a\,\underline{I_i}$

donc $\underline{I_1} - a^2\underline{I_2} = \underline{I_d} + \underline{I_i} - a^2(a^2\underline{I_d} + a\,\underline{I_i})$

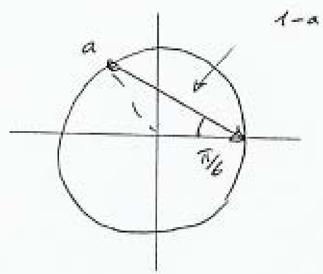
$= \underline{I_d}(1 - a^4) + \underline{I_i}(1 - a^3)$

$= \underline{I_d}(1 - a) + \underline{I_i}\underbrace{(1 - 1)}_{0}$

soit $\underline{I_1} - a^2\underline{I_2} = \underline{I_d}(1 - a)$

et $\boxed{\underline{V} = hR\,\underline{I_d}(1 - a)}$

7)



$\|1 - a\| = 2 \cdot 1 \cos\frac{\pi}{6} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

alors $\boxed{\|\underline{V}\| = hR\sqrt{3}\,\|\underline{I_d}\|}$

8) on permute $\underline{Z}$ et $\underline{Z}'$

8.1) alors $\underline{V_1} = h\,\underline{I_1}\,\underline{Z}'$ et $\underline{V_2} = h\,\underline{I_2}\,R$

soit $V_1 = -a^2 R\, h\,\underline{I_1}$

8.2) $\underline{V} = \underline{V_1} + \underline{V_2} = -a^2 R\, h\,\underline{I_1} + h\,\underline{I_2}\,R$

soit $\underline{V} = hR\,(\underline{I_2} - a^2\underline{I_1})$

$\underline{I_2} - a^2\underline{I_1} = a^2\underline{Id} + a\,\underline{Ii} - a^2\underline{Id} - a^2\underline{Ii}$

$= a\,\underline{Ii}\,(1-a)$

donc $\boxed{\underline{V} = a\,hR\,\underline{Ii}\,(1-a)}$

8-3). $\|a\| = 1 \qquad \|1-a\| = \sqrt{3}$

donc $\boxed{\|\underline{V}\| = hR\sqrt{3}\,\|\underline{Ii}\|}$

$$\begin{cases} \underline{I_1} = (1{,}72 - j\,2{,}58)\,A = 3{,}10\,e^{(-j\,56{,}3°)} \\ \underline{I_2} = (-2{,}50 - j\,2{,}13)\,A = 3{,}28\,e^{(-j\,139{,}6°)} \\ \underline{I_3} = (0{,}781 + j\,4{,}72)\,A = 4{,}78\,e^{(j\,80{,}6°)} \end{cases}$$

soit $\underline{I_1} + \underline{I_2} + \underline{I_3} = (0{,}001 + j\,0{,}01)\,A$. !

Calcul des composantes symétriques de courant

$\underline{Id} = \frac{1}{3}\left(\underline{I_1} + a\,\underline{I_2} + a^2\underline{I_3}\right)$

$= \frac{1}{3}\left(3{,}10\,e^{-j56{,}3} + 3{,}28\,e^{-j\underbrace{(240+139{,}6)}_{-j\,19{,}6}} + 4{,}78\,e^{j\underbrace{(-120+80{,}6)}_{-j\,39{,}4}}\right)$

$= \frac{1}{3}\left[1{,}72 - j\,2{,}58 + 3{,}09 - j\,1{,}1 + 3{,}69 - j\,3{,}04\right]$

soit $\underline{Id} = (2{,}83 - j\,2{,}24)\,A = 3{,}61\,e^{-j\,38{,}4°}$

$\underline{Ii} = \frac{1}{3}\left(\underline{I_1} + a^2\underline{I_2} + a\,\underline{I_3}\right)$

$= \frac{1}{3}\left(3{,}10\,e^{-j56{,}3} + 3{,}28\,e^{\underbrace{-j(120+139{,}6)}_{-j\,259{,}6}} + 4{,}78\,e^{\underbrace{-j(240-80{,}6)}_{-j\,159{,}4}}\right)$

$= \frac{1}{3}\left(1{,}72 - j\,2{,}58 - 0{,}594 + j\,3{,}23 - 4{,}47 - j\,1{,}68\right)$

$= -1{,}113 - j\,0{,}343 = 1{,}165\,e^{-j\,162{,}8°}$

<u>Vérification</u>: $\underline{I_1} = \underline{Id} + \underline{Ii}$

$= 2{,}83 - j\,2{,}24 - 1{,}11 - j\,0{,}34$

$= 1{,}72 - j\,2{,}58$ (C.Q.F.D.)