

$$2\omega_0 x''_1 = -2kx_1 - 3k(x_1 - x_2) - 2k(x_1 - x_3)$$

$$4\omega_0 x''_2 = -3k(x_2 - x_1) - 3k(x_2 - x_3) - 4kx_2$$

$$2\omega_0 x''_3 = -2kx_3 - 2k(x_3 - x_1) - 3k(x_3 - x_2)$$

$$2x''_1 + 7\omega_0^2 x_1 - 3\omega_0^2 x_2 - 2\omega_0^2 x_3 = 0$$

$$4x''_2 - 3\omega_0^2 x_1 + 6\omega_0^2 x_2 - 3\omega_0^2 x_3 = 0$$

$$2x''_3 - 2\omega_0^2 x_1 - 3\omega_0^2 x_2 + 7\omega_0^2 x_3 = 0$$

$$\begin{matrix} 1 & \left| \begin{array}{ccc} 2\omega_0^2 - 2\omega^2 & -3\omega_0^2 & -2\omega_0^2 \\ -3\omega_0^2 & 10\omega_0^2 - 4\omega^2 & -3\omega_0^2 \\ -2\omega_0^2 & -3\omega_0^2 & +7\omega_0^2 - 2\omega^2 \end{array} \right| \\ 2 & \\ 3 & \end{matrix} = 0$$

$$\begin{matrix} 1+2+3 & \left| \begin{array}{ccc} 2\omega_0^2 - 4\omega^2 & 4\omega_0^2 - 4\omega^2 & 2\omega_0^2 - 2\omega^2 \\ -3\omega_0^2 & 10\omega_0^2 - 4\omega^2 & -3\omega_0^2 \\ -2\omega_0^2 & -3\omega_0^2 & 7\omega_0^2 - 2\omega^2 \end{array} \right| \\ 2 & \\ 3 & \end{matrix} = 0$$

$$\left| \begin{array}{ccc} 1 & 2 & 3-1 \\ 2\omega_0^2 - 2\omega^2 & 4\omega_0^2 - 4\omega^2 & 0 \\ -3\omega_0^2 & 10\omega_0^2 - 4\omega^2 & 0 \\ -2\omega_0^2 & -3\omega_0^2 & 9\omega_0^2 - 6\omega^2 \end{array} \right| = 0$$

$$(9\omega_0^2 - 6\omega^2) \left[(2\omega_0^2 - 2\omega^2)(10\omega_0^2 - 4\omega^2) + 3\omega_0^2(4\omega_0^2 - 4\omega^2) \right] = 0$$

$$(9\omega_0^2 - 6\omega^2) \cdot 2(\omega_0^2 - \omega^2) (10\omega_0^2 - 4\omega^2 + 6\omega_0^2) = 0$$

$$2(9\omega_0^2 - 6\omega^2)(\omega_0^2 - \omega^2)(16\omega_0^2 - 4\omega^2) = 0$$

$$8(9\omega_0^2 - 6\omega^2)(\omega_0^2 - \omega^2)(4\omega_0^2 - \omega^2) = 0$$

$$\left[\begin{array}{l} \omega_0^2 = \omega^2 \\ \omega^2 = 4\omega_0^2 \\ \omega^2 = \frac{9}{2}\omega_0^2 \end{array} \right]$$

$$\omega^2 = \omega_0^2$$

$$\begin{array}{rrrcl} 5A & -3B & -2C & = 0 & 1 \\ -2A & -3B & +5C & = 0 & 2 \end{array} \quad 1-2 \Rightarrow 7A - 7C = 0 \quad \begin{array}{l} C = A \\ 1 \\ 1 \\ 1 \end{array}$$

$$\omega^2 = 4\omega_0^2$$

$$\begin{array}{rrrcl} -A & -3B & -2C & = 0 & 1 \\ -2A & -3B & -C & = 0 & 2 \end{array} \quad 1-2 \Rightarrow A - C = 0 \quad \begin{array}{l} C = A \\ B = -A \\ 1 \\ -1 \\ 1 \end{array}$$

$$\omega^2 = \frac{9}{2}\omega_0^2$$

$$\begin{array}{rrrcl} -2A & -3B & -2C & = 0 & (1) \\ -3A & -8B & -3C & = 0 & (2) \end{array} \quad 3 \times (1) - 2 \times (2) \Rightarrow B = 0 \quad \begin{array}{l} C = -A \\ 1 \\ 0 \\ -1 \end{array}$$

$$2x_1'' + 7\omega_0^2 x_1 - 3\omega_0^2 x_2 - \omega_0^2 x_3 = 0$$

$$4x_2'' - 3\omega_0^2 x_1 + 10\omega_0^2 x_2 - 3\omega_0^2 x_3 = \frac{F}{m} \text{ cos} \omega t$$

$$2x_3'' - 2\omega_0^2 x_1 - 3\omega_0^2 x_2 + 7\omega_0^2 x_3 = 0$$

$$(2\omega_0^2 - \omega_0^2)A - 3\omega_0^2 B - 2\omega_0^2 C = 0$$

$$-3\omega_0^2 A + (10\omega_0^2 - 4\omega_0^2)B - 3\omega_0^2 C = \frac{F}{m}$$

$$-2\omega_0^2 A - 3\omega_0^2 C + (2\omega_0^2 - \omega_0^2)C = 0$$

$$B_{det} = \begin{vmatrix} 2\omega_0^2 - \omega_0^2 & 0 & -\omega_0^2 \\ 0 & \frac{F}{m} & 0 \\ -\omega_0^2 & 0 & 2\omega_0^2 - \omega_0^2 \end{vmatrix} = \frac{F}{m} \left[(2\omega_0^2 - \omega_0^2)^2 - (\omega_0^2)^2 \right]$$

$$B_{det} = \frac{F}{m} (2\omega_0^2 - \omega_0^2)(\omega_0^2 - \omega_0^2)$$

$$A_{det} = \begin{vmatrix} 0 & -3\omega_0^2 & -\omega_0^2 \\ \frac{F}{m} & 0 & 0 \\ 0 & -3\omega_0^2 & 2\omega_0^2 - \omega_0^2 \end{vmatrix} = -\frac{F}{m} \left[-3\omega_0^2 (2\omega_0^2 - \omega_0^2 + \omega_0^2) \right]$$

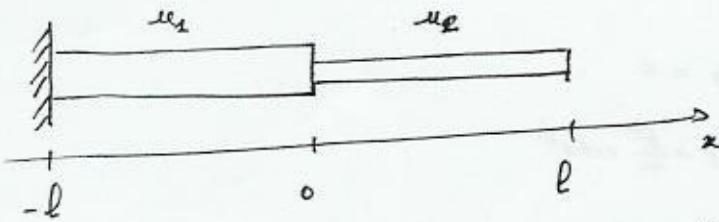
$$A_{det} = \frac{3F\omega_0^2}{m} (2\omega_0^2 - \omega_0^2)$$

$$C_{det} = \begin{vmatrix} 2\omega_0^2 - \omega_0^2 & -3\omega_0^2 & 0 \\ 0 & 0 & \frac{F}{m} \\ -\omega_0^2 & -3\omega_0^2 & 0 \end{vmatrix} = -\frac{F}{m} \left[-3\omega_0^2 (2\omega_0^2 - \omega_0^2 + \omega_0^2) \right]$$

$$C = A \quad \text{det} = \delta (2\omega_0^2 - \omega_0^2)(\omega_0^2 - \omega^2)(4\omega_0^2 - \omega^2)$$

$$A = C = \frac{3F\omega_0^2}{8m} \frac{(\omega_0^2 - \omega^2)(4\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)(2\omega_0^2 - \omega^2)}$$

$$B = \frac{F}{8m} \frac{5\omega_0^2 - \omega_0^2}{(\omega_0^2 - \omega^2)(4\omega_0^2 - \omega^2)}$$



$$\rho S \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} (E S \frac{\partial u}{\partial x}) = E S \frac{\partial^3 u}{\partial x^2}$$

$$u_1 = X_1 q$$

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2}$$

$$u_2 = X_2 q$$

$$\rho_1 X_1 q'' = E_1 X_1'' q \Rightarrow q'' = \frac{E_1 X_1''}{\rho_1 X_1} = -\omega_1^2$$

$$\rho_2 X_2 q'' = E_2 X_2'' q \Rightarrow q'' = \frac{E_2 X_2''}{\rho_2 X_2} = -\omega_2^2$$

$$k_1^2 = \omega_1^2 \frac{\rho_1}{E_1}$$

$$q'' + \omega^2 q = 0 \Rightarrow q = \alpha \cos \omega t + \beta \sin \omega t$$

$$k_2^2 = \omega_2^2 \frac{\rho_2}{E_2}$$

$$X_1'' + k_1^2 X_1 = 0 \Rightarrow X_1 = A_1 \cos k_1 x + B_1 \sin k_1 x$$

$$X_2'' + k_2^2 X_2 = 0 \Rightarrow X_2 = A_2 \cos k_2 x + B_2 \sin k_2 x$$

C.L.

$$u_1(0,t) = u_2(0,t) \Rightarrow X_1(0) \cdot q = X_2(0) \cdot q \Rightarrow X_1(0) = X_2(0) \quad (1)$$

$$N_1(0,t) = N_2(0,t) \Rightarrow E_1 S_1 X'_1(0) q = E_2 S_2 X'_2(0) q \Rightarrow E_1 S_1 X'_1(0) = E_2 S_2 X'_2(0) \quad (2)$$

$$u_1(-l,t) = 0 \Rightarrow X_1(l) q = 0 \Rightarrow X_1(-l) = 0 \quad (3)$$

$$N_2(l,t) = 0 \Rightarrow E_2 S_2 X'_2(l) \cdot q = 0 \Rightarrow X'_2(l) = 0 \quad (4)$$

$$(1) \quad A_1 = A_2$$

$$A_2 = A_1$$

$$(2) \quad E_1 S_1 k_1 B_1 = E_2 S_2 k_2 B_2$$

$$B_2 = \underbrace{\frac{E_1 S_1 k_1}{E_2 S_2 k_2}}_{\lambda} B_1$$

$$(3) \quad A_1 \cos k_1 l - B_1 \sin k_1 l = 0$$

$$(4) \quad k_2 (-A_2 \sin k_2 l + B_2 \cos k_2 l) = 0$$

$$\lambda = \frac{E_1 S_1}{E_2 S_2} \cdot \frac{\frac{\omega \sqrt{\rho_1}}{E_1}}{\frac{\omega \sqrt{\rho_2}}{E_2}} = \frac{S_1}{S_2} \sqrt{\frac{\rho_1 E_1}{\rho_2 E_2}}$$

$$A_2 \cos k_1 l - B_2 \sin k_1 l = 0$$

$$- A_2 \sin k_1 l + B_2 \cos k_1 l = 0$$

$$\boxed{\lambda \cos k_1 l \cos k_2 l \Rightarrow \sin k_1 l \sin k_2 l = 0} \quad \text{eq over fulgations frøres}$$

~~Det er ikke tilfældet~~

$$k_1 + k_2 = \omega \left(\sqrt{\frac{P_1}{E_1}} + \sqrt{\frac{P_2}{E_2}} \right)$$

ii $\frac{S_2}{S_1} = \sqrt{\frac{P_2 E_1}{P_1 E_2}} \Rightarrow \lambda = 1$

$$\cos(k_1 + k_2)l = 0 \Rightarrow (k_1 + k_2)l = (2n+1)\frac{\pi}{2}$$

$$(k_1 + k_2)_n = (2n+1)\frac{\pi}{2l}$$

$$\boxed{\omega_n = \frac{(2n+1)\frac{\pi}{2}}{\sqrt{\frac{P_1}{E_1}} + \sqrt{\frac{P_2}{E_2}}}}$$

$$\begin{aligned} \cos(k_1 + k_2)l = 0 &\Rightarrow \cos k_1 l \neq 0 \Rightarrow A_2 = B_2 \tan k_1 l \\ &\cos k_2 l \neq 0 \Rightarrow B_2 = A_2 \tan k_2 l \end{aligned} \quad \boxed{\tan k_1 l \tan k_2 l = 1}$$

$$B_2 = 1 \quad \begin{cases} A_2 = \tan k_1 l \\ A_2 = \tan k_2 l \\ B_2 = B_2 \end{cases}$$

$$\boxed{X_1 = \tan k_1 l \cos k_2 l + \sin k_2 l = \frac{\sin k_2 l (\tan k_1 l)}{\cos k_1 l}}$$

$$\begin{aligned} X_2 &= \tan k_2 l \cos k_1 l + \sin k_1 l = \cos k_1 l \cos k_2 l + \sin k_1 l \sin k_2 l \\ &= \frac{\cos k_1 l (\tan k_2 l)}{\sin k_2 l} \end{aligned}$$